Appendix C: Concurrent system analysis using Petri nets – an optimised algorithm for finding net invariants

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/* initialisation of the extended matrix */
   $\mathbf{C}$:= Incidence matrix $\subset Z^{mxn}$;
   $\mathbf{B}$:= Identity matrix $\subset Z^{nxn}$;

/* The extended matrix is the matrix obtained by adjoining B to the bottom of C giving a matrix of (m+n) x n dimension */

[phase 1]
while (there exists a non-zero element in $\mathbf{C}$) **do**
   [1.1] if (there exists a row h in C such that the sets
   $P^+ = \{j \mid c_{hj} > 0\}$, $P^- = \{j \mid c_{hj} < 0\}$
   satisfy $P^+ = \varnothing \vee P^- = \varnothing$)
   then
   [1.1.a] delete from the extended matrix all the columns of
   index $k \subset P^+ \cup P^-$
   else
   > [1.1.b] **if** (there exists a row h in $\mathbf{C}$ such that
   > $\mid P^+ \mid = 1 \vee \mid P^- \mid = 1$)
   > **then**
   > [1.1.b.1] let k be the unique index of column belonging to
   > P^+ (resp. to P^-);
   > > **for** $j \subset P^-$ (resp. $j \subset P^+$) **do**
   > > substitute to thecolumn of index j the linear
   > > combination of the columns indexed by k and j with
   > > coefficients $\mid c_{hj} \mid$ and $\mid c_{hk} \mid$ respectively
   > > **endfor**;
   > delete from the extended matrix the column of index k
   > **else**
   > [1.1.b.1] let h be the index of a non-zero row of $\mathbf{C}$;
   > let k be the index of a column such that $c_{hk} \neq 0$;
   > > **for** j such $j \neq k$, $c_{hj} \neq 0$ **do**
   > > substitute to the column of index j the linear
   > > combination of the columns of indices k and j
   > > with coefficients α and β defined as follows:
   > > > **if** $\text{sign}(c_{hj}) \neq \text{sign}(c_{hk})$ then $\alpha = \mid c_{hj} \mid$, $\beta = \mid c_{hk} \mid$
   > > > **else** $\alpha = -\mid c_{hj} \mid$, $\beta = \mid c_{hk} \mid$ **endif;**
   > > **endfor**;
   > delete from the extended matrix the column of index k
   > **endif**;
   endif;
endwhile;

[phase 2]

while (the matrix **B** contains a row of index h with
negative elements) **do**
let $P^+ = \{j \mid c_{hj} > 0\}$, $P^- = \{k \mid c_{hk} < 0\}$:

    if$(P^+ \neq \varnothing)$

    then for $(j,k) \subset P^+ \times P^-$ **do**

        operate a linear combination on the columns of indices
        j and k in order to get a new column having the
        h-th element equal to zero,
        divide this column by the GCD of its elements,
        append this column to the matrix B
        endfor;

    endif;
delete from B all the columns of index $k \subset P^-$
endwhile;
delete from B all the columns having non-minimal support.

The invariants appear as the columns of the matrix B, the bottom n rows of the
extended matrix.